HIP-2005-56/TH

Connection between Color Confinement and the Gluon Mass

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Abstract

It is argued that there are two phases in QCD distinguished by different choices of the gauge parameter. In one phase the color confinement is realized and gluons turn out to be massive, whereas in the other phase it ceases to be realized, but the gluons remain massless.

Foreword

It is a great honor for us to take part in the celebration of our dear friend, Professor Adriano Di Giacomo, on the occasion of his seventieth anniversary, and we feel a sense of privilege in dedicating this article to him. Adriano and we share a common interest in understanding color confinement (see, e.g. [1, 2] and references therein) so that we shall concentrate our attention to the exploration of the connection between confinement and the gluon mass in this article.

1 Quest for Massless Particles

Already in classical physics we know two kinds of fundamental interactions, namely, Coulomb and gravitational interactions. They have been recognized among many other phenomenological interactions by their long-range character obeying the inverse square law. In this connection it is interesting to observe that these two kinds of forces dominate in regions of different scales. At astronomical distances the gravitational forces dominate, but at microscopic distances the Coulomb forces take over. For instance, between two protons the gravitational force is weaker than the Coulomb force by 36 orders of magnitude. The rather vague border between these regimes is defined as distances at which the gravitational and electromagnetic forces balance. At the time of Millikan the size of an oil drop represented the border, but with the rapid progress of technology even the size of a linear motor car can be identified as a contemporary border.

Then a question is raised of why they dominate at different scales. In answering this question we may attribute the cause to the difference in the tensorial ranks of the fields mediating these interactions. The gravitational interactions are mediated by a second rank tensor field, whereas the electromagnetic interactions are mediated by a vector field.

In the former case both particle-particle and particle-antiparticle interactions are attractive, leading to the well-known universal attraction. In the latter case, however, only particle-antiparticle interactions are attractive, while particle-particle interactions are repulsive. It seems to be likely that there are no other forces obeying the inverse square law than those mentioned above. Thus in a macroscopic system like charges repel each other since there are no stronger attractive forces at such a scale and macroscopic systems tend to be electrically neutral. In a microscopic system, however, this is not the case since there are stronger attractive nuclear forces that overcome the Coulomb repulsion, and consequently we find multiply-charged nuclei. In this way, as the size of a system increases, the electromagnetic forces turn out to be less important and start to be taken over by gravity.

In quantum field theory, long-range forces obeying the inverse square law are generated by fields of massless quanta. Then it occurs to us that there would be no massless fields other than those mentioned above, since there is no experimental evidence for such a field

in the macroscopic world. In the microscopic world, however, the gluon - the quantum of the color gauge field - is a possible candidate for a massless particle, and we shall check this possibility in what follows.

The interaction between two electrically neutral systems is given by the van der Waals potential, proportional to r^{-6}. This shows that the electric fields generated by neutral systems can penetrate into the vacuum without any cut-off. This should be compared with the short-range character of strong interactions that are typically represented by the Yukawa potential between hadrons. Hadrons are color singlets and the color gauge field generated by them cannot penetrate into the vacuum beyond a certain limit of the order of the pion Compton wave length. Otherwise, the strong interactions would dominate the electromagnetic ones at all scales, in contradiction to our experience.

The above observation indicates that the gluons are confined within hadrons. Then it reminds us of the proton-electron model of the nucleus in the old days, in that it was very difficult in this model to confine light electrons inside a nucleus, since the uncertainty relation implies very high momentum, of the order of 100 MeV, for the electrons. In the case of strong interactions we are confronted with a similar difficulty again, since the uncertainty relation implies very high kinetic energies for the massless gluons, so that it seems very difficult, if not impossible, to confine them within a tiny hadron. A plausible way out of this difficulty would be to assume reasonably massive gluons, so that their kinetic energies turn out to be sufficiently low so as not to leak out of the hadron. On the other hand, when gluons are not confined there is no reason to believe why they should be massive.

Thus we predict a possibility of the presence of a connection between color confinement and the gluon mass and in the following sections we shall justify this intuitive reasoning theoretically [3].

2 Condition for Color Confinement

The problem of color confinement has been discussed elsewhere in detail [3]-[7] and we shall briefly state the condition for color confinement in our version. For this purpose we are going to introduce some notation, and start from the Lagrangian density of QCD:

$$\mathcal{L} = \mathcal{L}_{inv} + \mathcal{L}_{gf} + \mathcal{L}_{FP}, \tag{2.1}$$

where

$$\begin{aligned} \mathcal{L}_{inv} &= -\frac{1}{4}F^{\mu\nu} \cdot F_{\mu\nu} + i\overline{\psi}(\gamma^\mu D_\mu - m)\psi \ , \\ \mathcal{L}_{gf} &= -A^\mu \cdot \partial_\mu B + \frac{\alpha}{2} B \cdot B \ , \\ \mathcal{L}_{FP} &= -i\partial^\mu \overline{c} \cdot D_\mu c \end{aligned} \tag{2.2}$$

in the customary notation. We have suppressed the color and flavor indices in (2.2). The second term $\mathcal{L}_{gf}$ is the gauge-fixing term in which α denotes the gauge parameter and B the Nakanishi-Lautrup auxiliary field. The last term $\mathcal{L}_{FP}$ is the Faddeev-Popov (FP) ghost term, and the anticommuting scalar fields c and $\bar{c}$ denote the FP ghost fields. Only the first term $\mathcal{L}_{inv}$ is invariant under local gauge transformations, but the total Lagrangian is invariant under the global Becchi-Rouet-Stora (BRS) transformations [8].

Let us consider an infinitesimal gauge transformation of the gauge and quark fields and replace the gauge function either by c or by $\bar{c}$. Then they define two kinds of BRS transformations denoted by δ and $\bar{\delta}$, respectively:

$$\begin{aligned} \delta A_\mu &= D_\mu c, \quad \bar{\delta} A_\mu = D_\mu \bar{c}, && (2.3) \\ \delta \psi &= ig(c \cdot T)\psi, \quad \bar{\delta}\psi = ig(\bar{c} \cdot T)\psi. && (2.4) \end{aligned}$$

where the matrix T is introduced in the covariant derivative of ψ as

$$D_\mu \psi = (\partial_\mu - igT \cdot A_\mu)\psi \ , \tag{2.5}$$

For the auxiliary fields B, c and $\bar{c}$ the BRS transformations are determined by requiring the invariance of the local Lagrangian density, namely,

$$\delta \mathcal{L} = \bar{\delta} \mathcal{L} = 0 \ . \tag{2.6}$$

We shall not write them down explicitly since they are not relevant to the following arguments.

Then the equations of motion for the gauge field can be expressed with the help of BRS transformations as

$$\partial^\mu F_{\mu\nu} + gJ_\nu = i\delta\bar{\delta} A_\nu \ , \tag{2.7}$$

where J_ν denotes the color current density and g the gauge coupling constant. It is worth noting that all three terms in (2.7) are divergenceless separately, in particular

$$\partial^\nu (i\delta\bar{\delta} A_\nu) = 0 \ . \tag{2.8}$$

The generator of the BRS transformation δ is denoted by Q_B, then the physical states are defined as those states that are annihilated by applying Q_B. In this way we can safely eliminate the unphysical states of the indefinite metric. We interpret color confinement that colored particle states are unoservable, since they are not physical in the above sense. Then, as has been discussed before [3]-[7], the condition for color confinement is simply expressed by

$$C = 0 \ , \tag{2.9}$$

where the constant C is defined by

$$\partial^\nu \langle i\delta\bar{\delta} A^a_\nu(x), A^b_\sigma(y)\rangle = i\delta_{ab} C \partial_\sigma \delta^4(x-y) \ . \tag{2.10}$$

Here a and b are color indices and $\langle \cdots \rangle$ denotes the vacuum expectation value of the time-ordered product. Furthermore, with the help of the renormalization group (RG) and asymptotic freedom [9, 10], it has been shown that the conditions (2.9) is equivalent to

$$Z_3^{-1} = 0, \tag{2.11}$$

where Z_3 is the renormalization constant of the color gauge field. We shall come back to this subject later.

3 Connection between Color Confinement and the Gluon Mass

With the help of (2.7) we can write down an equation for two-point Green's functions of the form

$$\langle \partial^\lambda F^a_{\lambda\mu}(x), A^b_\nu(y)\rangle + \langle g J^a_\mu(x), A^b_\nu(y)\rangle = \langle i\delta\bar{\delta} A^a_\mu, A^b_\nu(y)\rangle \ . \tag{3.1}$$

We shall study the structure of the Fourier transforms of these Green's functions.

Let F_μ and G_ν be vector fields and introduce

$$\langle F_\mu(x), G_\nu(y)\rangle = \frac{-i}{(2\pi)^4}\int d^4k e^{ik\cdot(x-y)} T_{\mu\nu}(k), \tag{3.2}$$

and the Fourier transform of $\langle F_\mu, G_\nu\rangle$ is denoted by

$$T_{\mu\nu}(k) = \mathrm{FT}\langle F_\mu, G_\nu\rangle. \tag{3.3}$$

Then $T_{\mu\nu}$ can be expressed as a linear combination of two covariants:

$$T_{\mu\nu}(k) = -\frac{k_\mu k_\nu}{k^2 + i\epsilon} T_0(k^2) - \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2 + i\epsilon}\right) T_1(k^2). \tag{3.4}$$

Next we introduce two conditions:

1) Assume $\partial^\mu F_\mu = 0$, then we have

$$T_0(k^2) = T_0, \ \text{constant}. \tag{3.5}$$

2) Assume further that $Q|0\rangle = \langle 0|Q = 0$, where

$$Q = \int d^3x \ F_0(x), \tag{3.6}$$

then we have

$$T_0 = 0. \tag{3.7}$$

All three terms in (3.1) satisfy the constraint 1) and their Lehmann representations are given as follows. First, by taking account of the antisymmetry between subscripts λ and μ, we find

$$\mathrm{FT}\langle F_{\lambda\mu}, A_\nu\rangle = -i(k_\lambda g_{\mu\nu} - k_\mu g_{\lambda\nu})\left[\frac{R}{k^2+i\epsilon} + \int dm^2 \frac{\sigma_1(m^2)}{k^2-m^2+i\epsilon}\right] , \tag{3.8}$$

so that we obtain

$$\mathrm{FT}\langle \partial^\lambda F_{\lambda\mu}, A_\nu\rangle = -R\frac{k_\mu k_\nu}{k^2-i\epsilon} + (k^2 g_{\mu\nu} - k_\mu k_\nu)\int dm^2 \frac{\sigma_1(m^2)}{k^2-m^2+i\epsilon} . \tag{3.9}$$

Then, thanks to the condition 2), for the color charge Q we have

$$\mathrm{FT}\langle gJ_\mu, A_\nu\rangle = -(k^2 g_{\mu\nu} - k_\mu k_\nu)\int dm^2 \frac{\sigma_2(m^2)}{k^2-m^2+i\epsilon} . \tag{3.10}$$

Finally we have

$$\mathrm{FT}\langle i\delta\bar{\delta} A_\mu, A_\nu\rangle = -C\frac{k_\mu k_\nu}{k^2+i\epsilon} - (k^2 g_{\mu\nu} - k_\mu k_\nu)\int dm^2 \frac{\sigma_3(m^2)}{k^2-m^2+i\epsilon} . \tag{3.11}$$

Substituting these expressions for the three terms in (3.1), we obtain a simple relation:

$$R = C . \tag{3.12}$$

Here R represents the residue of the massless pole of the gluon propagator and $R = 0$ would mean the absence of the massless gluon. Thus the condition (2.9) for color confinement implies the gluons to be massive. On the other hand,when $C \neq 0$ we have massless gluons, since $R \neq 0$.

In this way we have found a connection between color confinement and the gluon mass. We call our attention, however, to the fact that all the expressions given in this section are unrenormalized ones, and we have to refine our arguments in the renormalized theory. This is done in the next section.

4 Renormalization Group

In order to justify the arguments developed in the preceding section we rely on the RG. The generator of this group is given by

$$\mathcal{D} = \mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - 2\alpha\gamma_V(g,\alpha)\frac{\partial}{\partial\alpha} , \tag{4.1}$$

where μ denotes the renormalization point with dimension of mass and γ_V is the anomalous dimension of the color gauge field.

We introduce a set of running parameters by

$$\begin{aligned}\overline{g}(\rho) &= \exp(\rho\mathcal{D})\cdot g,\\ \overline{\alpha}(\rho) &= \exp(\rho\mathcal{D})\cdot \alpha,\\ \overline{\mu}(\rho) &= \exp(\rho\mathcal{D})\cdot \mu = e^{\rho}\mu\ ,\end{aligned} \tag{4.2}$$

where ρ denotes the parameter of RG. The asymptotic limits of these running parameters are denoted by

$$\bar{g}(\infty) = g_\infty, \quad \bar{\alpha}(\infty) = \alpha_\infty\ . \tag{4.3}$$

Asymptotic freedom characteristic of non-Abelian gauge theories is expressed as

$$g_\infty = 0\ . \tag{4.4}$$

In solving the RG equations we introduce the ultraviolet cut-off so as to avoid divergences in the unrenormalized expressions, then in the asymptotic limit the running parameters and their functions reduce to bare or unrenormalized ones. The relationship (4.4), however, is obtained in the limit of the infinit cut-off.

Take, for instance, a running parameter $\bar{a}(\rho)$, then in the asymptotic limit we have:

$$\bar{a}(\infty) = a^{(0)}\ , \tag{4.5}$$

where $a^{(0)}$ denotes an unrenormalizable or bare expression and this relationship gives a boundary condition for $\bar{a}(\rho)$ when we solve the RG equation. On the other hand, the renormalized expression a is simply given by

$$\bar{a}(0) = a\ . \tag{4.6}$$

Let us consider two multiplicatively renormalizable running parameters $\bar{a}(\rho)$ and $\bar{b}(\rho)$ and assume that they satisfy RG equations of the form

$$\begin{aligned}\frac{d}{d\rho}\bar{a}(\rho) &= \gamma_a(\rho)\bar{a}(\rho)\\ \frac{d}{d\rho}\bar{b}(\rho) &= \gamma_b(\rho)\bar{b}(\rho)\ ,\end{aligned} \tag{4.7}$$

and the boundary condition

$$a^{(0)} = b^{(0)} \ \text{ or } \ \bar{a}(\infty) = \bar{b}(\infty)\ , \tag{4.8}$$

then $\bar{a}(\rho)$ can be expressed in terms of $\bar{b}(\rho)$ as

$$\bar{a}(\rho) = \bar{b}(\rho) + \int_\rho^\infty d\rho'[\bar{\gamma}_a(\rho') - \bar{\gamma}_b(\rho')]\bar{b}(\rho')\cdot \exp\int_{\rho'}^{\rho} d\rho''\bar{\gamma}_a(\rho'')\ . \tag{4.9}$$

The difference $a-b = \bar{a}(0)-\bar{b}(0)$ shows up in the renormalized expressions as the coefficient of the so-called Schwinger term [11]. The above result was obtained by combining (4.7) and (4.8), thereby keeping the cut-off finite. We assume, however, that (4.9) is valid also in the limit of an infinite cut-off and take it for granted in the renormalized version. Then, when one of them vanishes identically, for instance, $\bar{b}(\rho) = 0$, we may conclude that the other also vanishes, $\bar{a}(\rho) = 0$.

In the preceding section we have derived an equality

$$R^{(0)} = C^{(0)} \quad \text{or} \quad \bar{R}(\infty) = \bar{C}(\infty) \ , \tag{4.10}$$

and an application of the above arguments lead to the equivalence of the following two statements:

$$R = 0 \leftrightarrows C = 0 \ . \tag{4.11}$$

In this connection it should be emphasized that (4.11) was derived by comparing the residues of the massless spin zero poles in (3.1), whereas the equivalence of the two states

$$Z_3^{-1} = 0 \leftrightarrows C = 0 \tag{4.12}$$

mentioned in Section 2 can be derived by replacing the T-products by equal-time commutators in (3.1). In this way we have justified the connection between color confinement and the gluon mass.

In QED we have $C = 1$ so that there is no charge confinement and the photon is massless. This is a typical example to show the relevance of the connection discussed in this paper.

Next we shall check when the confinement condition is satisfied and for this purpose we consult the following identity [3]-[7]:

$$Z_3^{-1} = \frac{\alpha}{\alpha_\infty} \ . \tag{4.13}$$

The parameter α_∞ can assume the following three values:

$$\alpha_\infty = -\infty, \ 0, \ \alpha_0 \ , \tag{4.14}$$

where

$$\alpha_0 = \frac{1}{3}(13 - \frac{4}{3}N_f) \ . \tag{4.15}$$

N_f denotes the number of quark flavors, and for simplicity we shall confine ourselves to the simple case of positive α_0, then we have [12, 13]

$$Z_3^{-1} = \begin{cases} 0 \ , & \text{for} \quad \alpha \le 0 \\ \alpha/\alpha_0 \ , & \text{for} \quad \alpha > 0 \end{cases} \tag{4.16}$$

corresponding to

$$\alpha_\infty = \begin{cases} -\infty \ , & \text{for} \quad \alpha < 0 \\ 0 \ , & \text{for} \quad \alpha = 0 \\ \alpha_0 \ , & \text{for} \quad \alpha > 0 \end{cases} \tag{4.17}$$

Thus we may conclude that color confinement is realized for $\alpha \leq 0$ and the gluon turns out to be massive, whereas it is not realized for $\alpha > 0$ and the gluon remains massless. It might occur to us that physical results should be independent of the choice of the gauge parameter, but it is not always the case.

In showing the equivalence between theories of two different gauge parameters, say α and $\alpha + \Delta\alpha$, we usually refer to power series expansions of Green's functions in $\Delta\alpha$. If this series converges, we may infer the gauge independence, but eventually $\Delta\bar{\alpha}(\rho)$ diverges in the limit $\rho \to \infty$. This is really the case when $\alpha > 0$ and $\alpha + \Delta\alpha < 0$ or $\alpha < 0$ and $\alpha + \Delta\alpha > 0$. Then we have to conclude that there are two distinctive phases in QCD depending on the signature of the gauge parameter although it contradicts our naïve belief. It should also be mentioned that the transition between these two phases cannot take place since the gauge parameter is not an adjustable dynamical variable and only the confinement phase seems to be realized in nature.

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